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Design Tukey's Control Chart and mix with CUSUM Control Chart

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Abstract

Statistical process control is a collection of valuable tools for detecting alteration in a process. It has wide application in many areas field and other fields where variation is being monitored. The variation may be a natural cause variation or a particular cause variation.
Statistical process control deals with the monitoring process to detect disturbances in the process. These disturbances may be from the process mean or variance. This study proposes efficient charts for detecting early shifts in dispersion parameters by applying the Fast Initial Response feature.
We propose and compare the performance of different cumulative sum (CUSUM)control charts for phase II monitoring of location based on mean and median. The (CUSUM) control chart, which is a method of data analysis based on John Tukey's principles control chart (TCC), is used to compare the proposed charts with their existing counterparts is used to evaluate new charts to existing charts using performance measures such as average run length, the standard deviation of run length, additional quadratic loss, relative average run length, and performance comparison .
The proposed charts detect early shifts in the process dispersion faster and have better overall. This article is a similar effort to design an improved charting structure in the form of mixed or using Tukey -CUSUM chart together, to show the process control chart., and drawing the Average Run Length ARL value.

Introduction

Statistical Process Control (SPC) refers to a set of several practical methods used for an effective process monitoring. Control charts are efficient monitoring tools and they appear as graphical displays with specific lower and upper bounds to monitor the quality of ongoing processes.
Cumulative sum control charts were first proposed by Page (1954) and have been used by many authors. have been found to be more effective in identifying minor variations in the mean of a process than Shewhart charts, indicates that they are superior to Shewhart control charts, and used to take care of the monitor small shifts in the process mean [3] ,[10]&[11]. It uses the cumulative sum of deviations up and down from a target.
The cumulative sum control chart is formed by plotting the quantity as:

$$C_i = \sum_{j=1}^{n}(X_j - \mu_0) \qquad \ldots\ldots 1$$

Tabular CUSUM Procedure

The CUSUM requires two parameters: In sigma units, a reference value (k) is provided. K is frequently adjusted to half the shift in sigma units to be detected. Default k=0.5, which corresponds to a one sigma change. The sigma units are often used to specify the decision limit (h). h=5 is the default value.

The parameter H defines as H=hσ and K= kσ where σ is the standard deviation of the samples used in forming the CUSUM [2],[9],[12].
CUSUM can be shows as two ways: the tabular CUSUM and the V-mask CUSUM.
The tabular CUSUM works by accumulating derivations from μ0 with a two-sided we plot the involves two statistics , C_i^+ and C_i^- are called one-sided upper and lower CUSUM. The upper sided above the target is C_i^+ and the lower sided below the target value is C_i^- respectively., initially C_i^+ and C_i^- and are set to zero. They are computed as follows [3], [11], [14].

$$C_i^+ = max[0, X_i - (\mu_0 + K) + C_{i-1}^+] \quad \ldots 2$$
$$C_i^- = max[0, X_i - (\mu_0 + K) + C_{i-1}^-]$$

Where Xi denote the i-th observation, and μ_0 is the target value.
Where The Starting values are $C_0^+ = C_0^- = 0$ …. 3
Where K is usually called the reference value the allowance, or the slack value is often chosen about halfway between the target and the out-of-control value of the mean that we are interested in detecting quickly.
Then K is one-half the magnitude of the shift:

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2} \quad \ldots 4$$

Where μ_1 the actual mean of the process and μ_0 the target mean of the number.
Where C_i^+ and C_i^- accumulate deviations from the target value μ_0 that are greater than K, with both quantities reset to zero on becoming negative. For a one-sided If C_i^+ or C_i^- exceed the decision interval H, the process is considered to be out of control.[1][3][14]

TABULAR CUSUM PROCEDURE [3] &[9]

The Tabular CUSUM is designed by choosing values for the reference value K and the decision interval H. These parameters be selected to provide a good average run length performance.
Average Run Length (ARLs) due to Brook and Evans (1972) that is based on approximating transitions from the in control to the out-of-control. Quality control schemes are usually evaluated by their ARL The Average Run Length of Cumulative Sum Control Charts; The ARL should be significant when the process, is operating on target. The operation of obtaining samples to use with a cumulative sum (CUSUM) control chart consists of taking samples of size n and plotting the cumulative sums
ARL approximation given by Siegmund (1985) because of its simplicity [1], [3], [9] &[14].
ARL Approximation for upper-side CUMUM ARL+ and lower –side CUMSE ARL- with parameters *h* and *k*, are given by:

$$ARL = \frac{EXP(-2\Delta b) + 2\Delta b - 1}{2\Delta^2} \quad \ldots 5$$

For $\Delta \neq 0$, where $\Delta = \delta * - k$ for the upper one-sided CUSUM $C+$, $\Delta = -\delta * - k$ for the lower one-sided CUSUM $C-$, $b = h + 1.166$, and If $\Delta = 0$, one can use ARL $= b^2$

The formula for the ARL

$$ARL = \frac{\exp(-2(-\delta^{\mu} - k)(h + 1.166)) + 2(-\delta^{\mu} - k)(h + 1.166) - 1}{2(-\delta^{\mu} - k)^2} \quad \ldots 6$$

To obtain the ARL of the two-sided CUMSE from the ARLs of the two one-sided statistics say, ARL+ and ARL- ;

$$\frac{1}{ARL} = \frac{1}{ARL^+} = \frac{1}{ARL^-} \quad \ldots 7$$

Tukey's Control Chart (TCC) [2],[5]&[7].

TCC is remarkably easy to construct When a large number of data points are examined; the extreme observation is averaged with many other observations, reducing the influence of outliers on control limits.

When working with a limited number of data (e.g., 15), the outlier can significantly affect the tightness of control limits, even when working with a small data set.
averages depending on the small data sets. Because medians may be computed by looking at the data in sequence, John Tukey used a famous statistician alternative way, with medians and exploratory data analysis. Tukey suggested confidence intervals for medians.
Since medians are not affected by outliers, they are ideal for the analysis of small data sets.
To define the control chart, we used John Tukey's techniques for generating confidence intervals for medians. This style of the graph is known as a "Tukey's Control Chart." Tukey's Control Chart provides several benefits. The first assumption of Tukey's Control Chart makes no about the data distribution. It is not assumed that the observations are Normal in the distribution or follow any other specified distribution. As such, Tukey's Control Chart can be applied to any interval-based data.
Second, with small databases, Tukey's Control Chart can be used to graph the upper control limit (UCL) and the lower control limit (LCL). However, practically speaking, it can be more precision most improvement with more data to construct the control chart depending on.
The actual number of data points.
We depend on the consequence of waiting and collecting more data versus employing too little data and making an error in judgment [4],[6]&[11].
Alemi (2004) used the Tukey control chart (TCC) as an excellent alternative to the X/MR chart for individual process monitoring, when data follow the skewed distribution and have some outliers.
Borckardt et al. (2005, 2006) operated the Tukey chart with serially dependent data. Several authors investigated the performance of the Tukey chart under conditions [11],[12]&[15].
The Tukey control chart limits (central line (CL), lower control limit (LCL), and upper control limit (UCL)) are given as:

$$LCL = Q1 - L(IQR)\ ,\ \ CL = Q2,\ \ UCL = Q3 + L(IQR) \qquad \ldots.\ 8$$

The parameter L is usually set as L = 1.5 Where Q1 and Q3 are quartile
where :

$$Q1 = F^{-1}(0.25),\ Q3 = F^{-1}(0.75) \qquad \ldots.\ 9$$

and IQR = Q3 − Q1. Under the normal distribution with population mean μ and population standard deviation δ, [10],[11]&[13]:

The ARL Calculation for the Tukey's Control Chart [8],[10],[11]&[12].
When the control limits of the TCC have been set, the in-control and out-of-control average run lengths can be calculated. Assuming that the process is in control, then the ARL_0 for the TCC can be calculated as: follows:

$$ARL_0 = \frac{1}{1-\int_{F^{-1}(0.25)-kIQR}^{F^{-1}(0.75)+kIQR} f(x)dx} = \frac{1}{\alpha} \qquad \ldots.\ 10$$

If the process is out of control or the mean shift of δσ occurs, then the ARL_1 for the TCC can be computed as following;

$$ARL_1 = \frac{1}{1-\int_{F^{-1}(0.25)-(kIQR)-\delta\sigma}^{F^{-1}(0.75)+(kIQR)-\delta\sigma} f(x)dx} = \frac{1}{1-\beta} \qquad \ldots.\ 11$$

Where α is the probability of Type I error, β s the probability of Type II error, and f(x)is the probability density function (p.d.f) and F(X)is a cumulative distribution function of a random variable of X [10].
where L denotes the control limits coefficient that may be fixed at pre-specified ARL_0 and Q1, Q2 and Q3 are first, second and third quartiles respectively, and IQR *is* Inter-Quartile Where IQR is Q3-Q1:

CUSUM-TCC [7] ,[8]&[11].

The CUSUM chart plots the cumulative sums of the deviations of sample values from a target value. The cumulative sum, up to and including the ith sample, is denoted by Ci and is given in the form of two statistics C_i^+ and C_i^- as:

$C_i^+ = \max[0, X_i - Q_3^s - K + C_{i-1}^+]$ ….. 12

$C_i^- = \max[0, Q_1^s - X_i - K + C_{i-1}^-]$ ….. 13

where Q1 and Q3 are adjusted Q1 and Q3 respectively defined as:

$$Q_3 = (Q_3 - 0.75Q_3) + 0.5Q_2 + 0.25Q_1$$
$$Q_1 = (Q_1 - 0.75Q_1) + 0.5Q_2 + 0.25Q_3 \quad ….. 14$$

Q3 and Q1 are adjusted quartile values

.

NUMERICAL ILLUSTRATION

This paper highlights the positive measurement features of the above proposed Tukey and Tukey CUSUM control chart limit. Then drawing charts of both control charts to compare the numerical result analysis and charts. in this case used the (40) samples with a sample size of (4) of the (PH) components of from the chemical analysis from chemical laboratory of (Directorate Preventive health – Sulaimaniyi), and assuming that h= 4 or h=5 and k=0.5his provide that the CUSUM control chart limit. Then drawing charts of both control charts to compare the numerical result analysis an has good ARL.

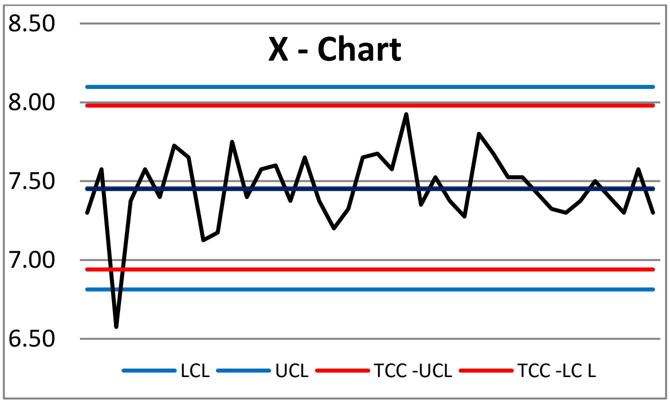
The total measurement number is (160). As shown in the index table.

By using the data from the appendix table we calculating the UCL and LCL for X-Shewhart and TCC control chart, as shown in table (1). The UCL for Shewhart is (8.10), and for TCC (7.98), the UCL in Shewhart smaller than TCC, the Range between UCL and LCL of TCC is (1.04), Shewhart is(1.28). The range of Shewhart is greater than TCC, as shown in Fig (10).

Table(2) contains the value of ARL of Shewhart with a different value of sigma (3,2,1) as shown in Fig (2)., and table(3) is for the value of ARL of TCC and Shewhart.it is seen that all matter of ARL TCC is smaller than ARL Shewhart. As show in fig(3).(used R-program and Stat graphics)

Table(1) UCL and LCL

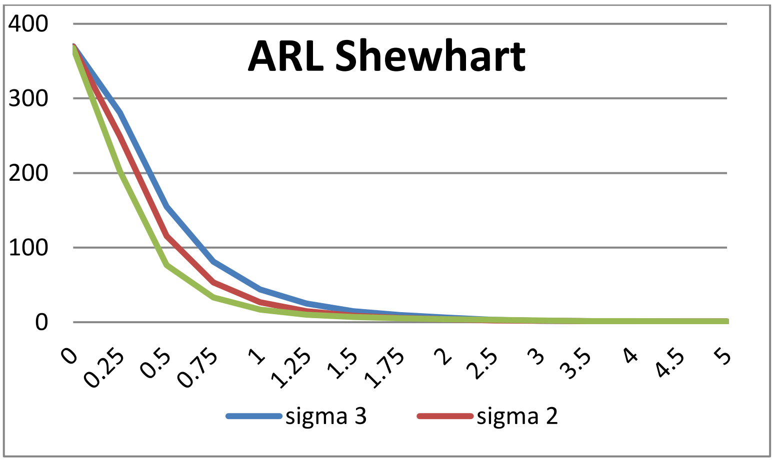
	Shewhart	TCC
UCL	8.10	7.98
LCL	6.81	6.94
Average	7.46	
Range	1.28	1.04
Median		7.41



Fig(1) TCC and X-Shewhart Chart

Table(2) ARL Of X-Shewhart

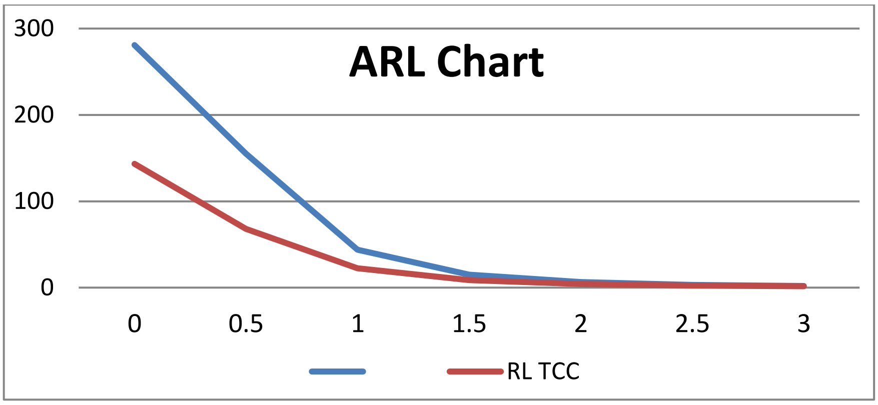
Shift	sigma 3	sigma 2	sigma 1
0	370	370	370
0.2	308.11	283.86	245.2
0.4	199.88	159.14	113.49
0.6	119.56	84.25	54.33
0.8	71.49	46.16	29.05
1	43.86	26.78	17.39
1.2	27.8	16.53	11.5
1.4	18.23	10.84	8.28
1.6	12.38	7.52	6.38
1.8	8.69	5.49	5.19
2	6.3	4.2	4.38
2.2	4.72	3.35	3.79
2.4	3.64	2.76	3.33
2.6	2.9	2.34	2.94
2.8	2.38	2.03	2.6
3	2	1.8	2.3
3.2	1.73	1.62	2.03
3.4	1.53	1.48	1.8
3.6	1.38	1.37	1.61
3.8	1.27	1.28	1.45
4	1.19	1.21	1.33
4.2	1.13	1.15	1.24
4.4	1.09	1.11	1.16
4.6	1.06	1.08	1.11
4.8	1.04	1.05	1.08
5	1.02	1.03	1.05



Fig(2) ARL chart Of X-Shewhart Chart

Table(3)ARL Of X-Shewhart and TCC

Shift mean	ARL Shewhart	ARL TCC
0	280.86	143.34
0.5	155.08	68.1
1	43.86	22.29
1.5	14.96	8.66
2	6.3	4.12
2.5	3.24	2.37
3	1.9	1.62

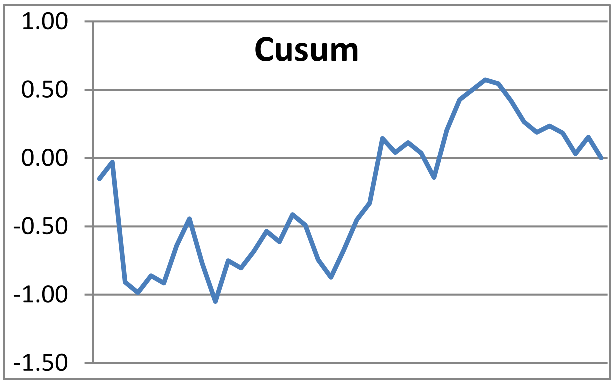


Fig(3) ARL Chart of X-Shewhart and TCC

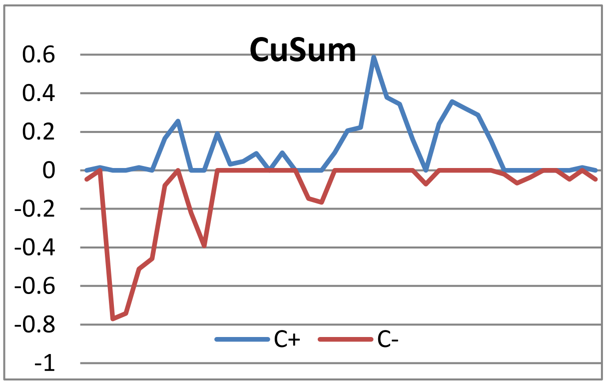
From appendix table data, Determine the Average, CUSUM C+, C- and Quantities N+, Quantities N-as shown in table (4). The Fig (4,5,6) shows that the control charts for (CUSUM C+, C- and N+, N-). Fig (4) the upper and lower control limit of CUSUM ,the limits between (0.1 and -0.1) from the CUSUM chart is seen that the value increasing up to a point (19), and (18) points are positive +. Fig (2) shows that most points are rounding the upper limit and (15) points out of the upper limit. Fig(3) of Quantities N+ and N- shows that the maximum rank for N+ is (7), and for N- is (5).

Table (4) Calculating CuSum and Tukey-CuSum

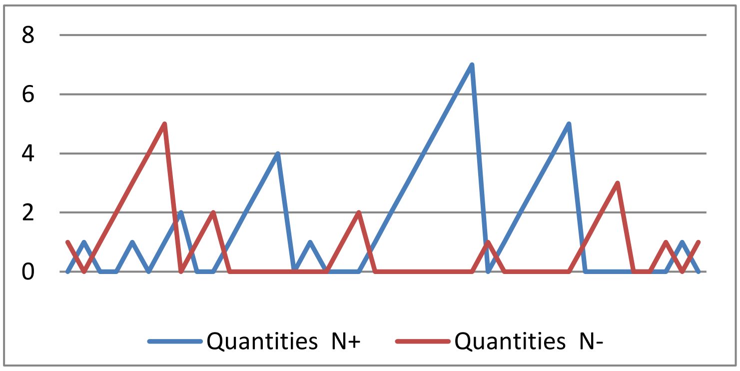
	CuSum						Tukey				
#	Averag	Cusum	C+	Quantits N+	C-	Quantits N-	Tuyke - Cusum	C+	Quantits N+	C-	Quantits N-
1	7.3	-0.15	0	0	-0.05	1	-0.11	0.00	0	0	0
2	7.58	-0.03	0.02	1	0	0	0.16	1.00	1	0.00	0
3	6.58	-0.91	0	0	-0.77	1	-0.84	0.00	0	0.84	1
4	7.38	-0.98	0	0	-0.74	2	-0.04	0.00	0	0.88	2
5	7.58	-0.86	0.02	1	-0.51	3	0.16	-2.00	0	1.71	3
6	7.4	-0.91	0	0	-0.46	4	-0.01	0.00	0	1.73	4
7	7.73	-0.64	0.17	1	-0.08	5	0.31	-4.00	1	0.00	0
8	7.65	-0.44	0.26	2	0	0	0.24	-2.00	2	0.00	0
9	7.13	-0.77	0	0	-0.22	1	-0.29	0.00	0	0.29	1
10	7.18	-1.05	0	0	-0.39	2	-0.24	0	0	-0.29	2
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
35	7.38	0.19	0	0	-0.04	3	-0.04	0	0	0	0
36	7.5	0.24	0	0	0	0	0.09	0	0	0	0
37	7.4	0.18	0	0	0	0	-0.01	0	0	0	0
38	7.3	0.03	0	0	-0.05	1	-0.11	0	0	0	0
39	7.58	0.15	0.02	1	0	0	0.16	0.05	1	0	0
40	7.3	0.00	0	0	-0.05	1	-0.11	0	0	0.00	0



Fig(4) CuSum Chart

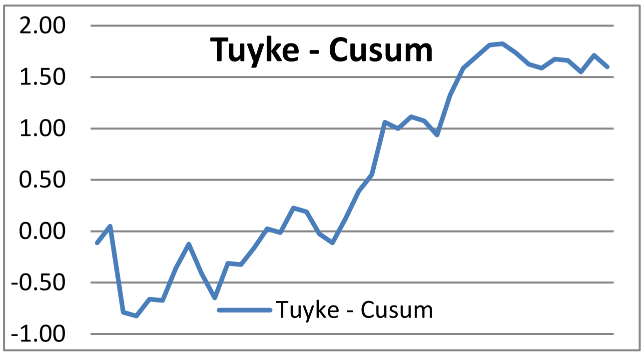


Fig(5) UCL and LCL of CuSum Chart

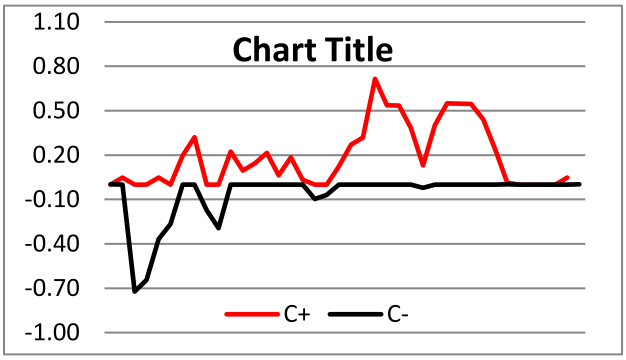


Fig(6) Quantities N+ ,N – of CuSum Chart

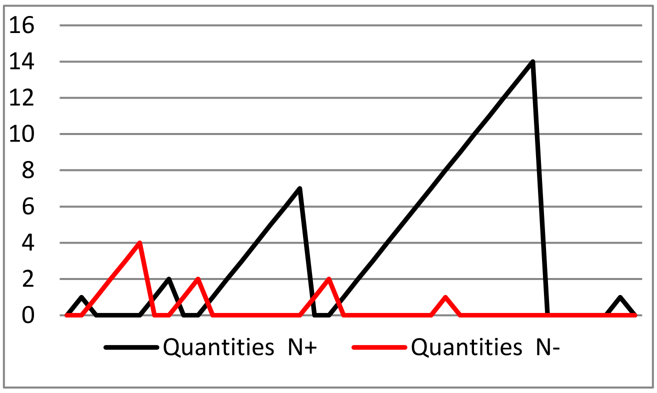
By using the Eq. (13,14) calculating the TCC- CUSUM as shown in table (4), the upper and lower control limit is(0.12, -0.12) it seen that most of the points out of the range and increasing to upper show in Fig (7) .The rank of Quantities N+ s (14), as shown in Fig (9) and Fig.(10), Shows that the chart of two (Tuyke and CUSUM), seen in the Tuyke chart, increasing to upper.



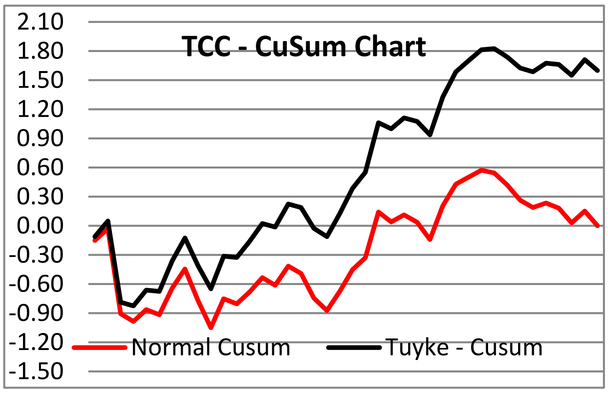
Fig(7) Tuyke–Cusum Chart



Fig(8) UCL and LCL of Tuyke CuSum Chart



Fig(9) Quantities N+ ,N – of Tuyke Chart

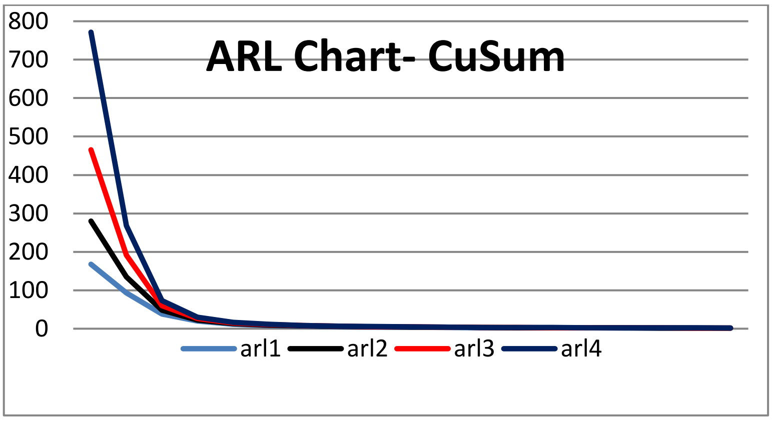


Fig(10) CuSum and TCC-CuSum chart

Table(5) and Fig (11). Shows that the different value of ARL- CUSUM for different values of parameter (H=4, 4.5, 5, 5.5) and k=0.5. Table(5) and Fig(12) shows the value of ARL of Cusum and TCC-CUSUM if the parameter k=0.5 and H=5, from table(6) it seen that the value of ARL of TCC-CUSUM are smaller than ARL- CUSUM.

Table(5) ARL of CuSum

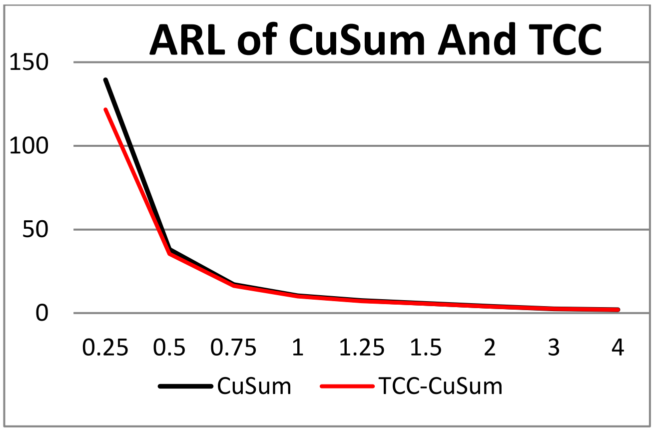
	k=0.5 H=4	k=0.5 H=4.5	K=0.5 H=5	k=0.5 H=5.5
Shift mean	arl1	arl2	arl3	arl4
0	167.68	279.97	465.44	771.55
0.2	93.42	134.74	191.28	268.28
0.4	38.58	48.44	59.82	72.9
0.6	19.45	22.76	26.23	29.84
0.8	11.94	13.54	15.16	16.79
1	8.38	9.38	10.38	11.37
1.2	6.42	7.13	7.84	8.56
1.4	5.2	5.75	6.31	6.86
1.6	4.37	4.83	5.28	5.74
1.8	3.78	4.17	4.55	4.94
2	3.34	3.68	4.01	4.34
2.4	2.73	2.99	3.26	3.52
2.6	2.52	2.75	2.98	3.22
2.8	2.34	2.55	2.76	2.98
3	2.19	2.38	2.57	2.77
3.5	1.92	2.08	2.23	2.38
4	1.71	1.88	2.01	2.12
4.5	1.5	1.7	1.86	1.97
5	1.31	1.5	1.69	1.85



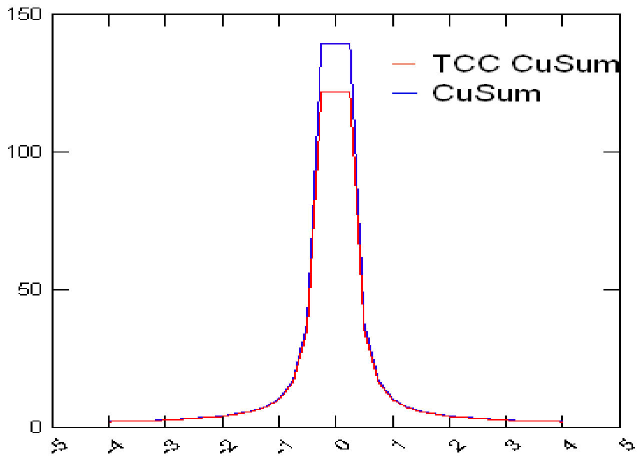
Fig(11) ARL of CuSum Chart

Table (6) ARL of CuSum and TCC-CuSum

Shift mean	CuSum k=0.5 ,H=5	TCC-CuSum
0.25	139.49	121.72
0.5	38	35.37
0.75	17.05	16.31
1	10.38	9.89
1.25	7.39	7.15
1.5	5.75	5.56
2	4.01	3.87
3	2.57	2.49
4	2.01	1.95



Fig(12) ARL of CuSum and TCC-CuSum



Fig(13) ARL of CuSum and TCC-CuSum

Conclusion

This article focuses on using the TCC technique and comparing the TCC to the Shewhart method to develop and construct quality. To create and construct a quality control chart. The TCC technique is more successful than the Shewhart method, as seen by the control chart and ARL value. The value of ARL of TCC shows that all matter of ARL TCC is smaller than ARL Shewhart.

This research computed the upper and lower limits and C+, C-, N+, N-, and N- for the CUSUM and TCC CUSUM Charts. It shows that the TCC CUSUM design is more sensitive and that the process mean shifted slightly. The ARL of TCC CUSUM is smaller than the ARL of CUSUM.

As can be seen, this design works best when the data is skewed, but it also works well when the data is asymmetric. CUSUM-TCC seems to be a feasible alternative in several instances.

Appendix Table Data

Data					
#	X1	X2	X3	X4	Averag
1	7.4	7.3	7.8	6.7	7.3
2	7.6	8.5	7.3	6.9	7.58
3	7.5	7	4.7	7.1	6.58
4	8.3	7.3	6.8	7.1	7.38
5	7.9	7.8	7.4	7.2	7.58
6	8.1	7	7.1	7.4	7.4
7	7.3	7.6	7.6	8.4	7.73
8	7.1	8	8	7.5	7.65
9	7.6	6.9	7.1	6.9	7.13
10	7.6	6.9	7	7.2	7.18
•	•	•	•	•	•
•	•	•	•	•	•
•	•	•	•	•	•
35	7.5	7.6	7.5	6.9	7.38
36	7.5	7.4	7.6	7.5	7.5
37	7.8	7.2	7.5	7.1	7.4
38	7.4	7.6	7.2	7	7.3
39	7	7.1	9.1	7.1	7.58
40	7.1	6.9	8.1	7.1	7.3

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